

Material Covered

Test 2 is cumulative and covers through Section 6.4.5 (and the portions of the appendices we have used). The emphasis will be on things covered since Test 1, but you should not have forgotten things that were covered on Test 1.

Note that there is a summary at the end of each chapter that briefly reviews the most important topics of the chapter.

Things to be sure you review

This is not intended to be an exhaustive list, but the items below are important things to review.

1. Statistical models and distributions

- You will be provided with the tables of distributions, so you don't need to memorize all that information, but you should know how to work with it.
- You should recognize situations when various favorite distribution families are (potentially) good models.
- Basic probability rules and the cdf method
- Recognizing a distribution from its kernel
- Comparing data to a model (qq plots, density overlay on a histogram, etc.)
- Mixture distributions (as we used in the Old Faithful example)

2. Point Estimates for Parameters in a Model

- Method of moments estimation
- Definition of likelihood function
- Finding MLEs by maximizing the likelihood function (analytically and/or numerically)
- For numerical results, you should know how to (a) create log-likelihood functions in R and (b) how to use the `maxLik` package to optimize.

3. Likelihood Ratio Tests

- Definition and calculation of likelihood ratio statistic (λ) and the normalized statistic $W = -2 \log(\lambda)$.
- (Approximate) distribution of likelihood ratio statistic (using $\dim(\Omega)$ and $\dim(\Omega_0)$ to calculate degrees of freedom).
- p-values via simulation.
- Tests based on tabularized or binned data.
 - maximum likelihood and Pearson statistics
 - definition of expected counts
 - goodness of fit tests
 - inference for two way tables

4. Confidence Intervals

- Score and information functions
- Quadratic approximation to the log-likelihood function
- Wald and likelihood ratio confidence intervals; computing (approximate) standard error using the observed information.
- Inverting p-values to obtain confidence intervals

5. Preparation for Linear Models and Linear Algebra

- The method of least squares and its relationship to vectors.
- Dot products, projections, orthogonal vectors, spans, unit vectors.
- Matrix multiplication, matrix transpose, matrix inverse
- Chi-squared distributions as sums of independent squared $\text{Norm}(0, 1)$.
- “The Picture” and how it relates to fitting a linear model.

6. Point Estimates for Parameters in a (Simple) Linear Model

- Maximum likelihood estimates.
- Least squares via calculus.
- Least squares via projections of vectors.
- Least squares via matrix operations.
- (\bar{x}, \bar{y}) is on regression line.
- $\hat{\beta}_1 = r \frac{s_y}{s_x}$.

7. Applying linear models

- Model formulas and interpretation of parameters.
- Fitting linear models with `lm()`.
- Interpreting parameter estimates.

8. Inference for linear models

- $\hat{\beta}_1 \sim \text{Norm}\left(\beta_1, \frac{\sigma}{|\mathbf{x} - \bar{\mathbf{x}}|}\right)$.
- $\hat{\beta}_0 \sim \text{Norm}\left(\beta_0, \sigma \sqrt{\frac{1}{|\mathbf{x}|^2} + \frac{\bar{\mathbf{x}}^2}{|\mathbf{x} - \bar{\mathbf{x}}|^2}}\right)$.
- $\frac{(n-2)S^2}{\sigma^2} \sim \text{Chisq}(n-2)$.