Probability Review

Math 252

1. Suppose $p(A) = 0.4$, $p(B) = 0.3$, and $P(A \mid B) = 0.2$.
   a. Are A and B independent? How do you know?
   b. Are A and B mutually exclusive? How do you know?
   c. What is $p(A \cap B)$?
   d. What is $p(A \cup B)$?
   e. What is $p(B \mid A)$?
2. Can two events be both mutually exclusive and independent? If so, give an example. If not, explain why not.
3. Let $X \sim \mathsf{Binom}(n, p)$. What is $\mathrm{E}(X)$?

   [Hint: Use the Linearity Theorem. How can you write X as a sum?]
4. What is the expected number of diamonds in a 5-card hand dealt from a standard deck?
5. A state issues license plates with 3 letters followed by 3 digits. I random license plate configuration is selected, what is the probability that one of the letters or digits is repeated?
6. If you roll 5 4-sided dice, what is the expected number of 4s rolled?
7. If you roll 5 standard dice, what is the expected number of distinct numbers rolled? [Example: if you roll 1, 1, 3, 4, 5; then there only 4 distinct numbers.]
8. Can you prove the linearity theorem? [For part b, you may restrict your attention to a sum of two random variables.]