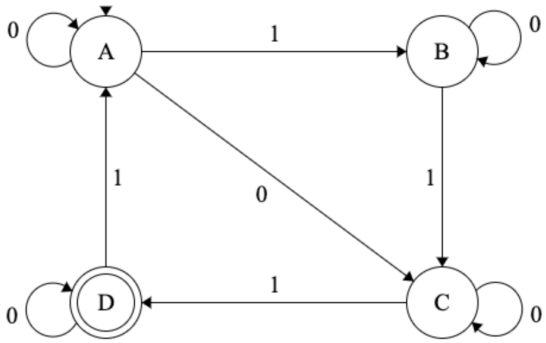
4.3 Recognizing Regular Languages with NFAs

1. Show that the empty language ($\emptyset$) is recognized by an NFA.
2. Show that the language $\{\lambda\}$ is recognized by an NFA.
3. Show that if $a \in I$, then the language $\{a\}$ is recognized by an NFA.
4. An NFA is said to have a **lonely start state** if there are no transitions into the start state.
   a. Contruct an NFA with a lonely start state that is equivalent to the NFA below.



   b. Explain why every NFA is equivalent to an NFA with a lonely start state. That is, describe how to convert any NFA into an equivalent NFA with a lonely start state.
5. Show that if A and B are each recognized by an NFA, then AB is, too.

   [Hint: Let N_A and N_B be the NFAs that recognize A and B. You may assume each has a lonely start state if that is useful. How can you use them to build a new NFA that recognizes AB?]
6. Show that if A and B are each recognized by an NFA, then $A \cup B$ is, too.

   [Hint: Let N_A and N_B be the NFAs that recognize A and B. You may assume each has a lonely start state if that is useful. How can you use them to build a new NFA that recognizes $A \cup B$?]
7. Show that if A is recognized by an NFA, then A^* is, too.

   [Hint: Let N_A be the NFA that recognizes A. You may assume it has a lonely start state if that is useful. How can you use it to build a new NFA that recognizes A^*?]
8. Explain how 1–7 above show that every regular language is recognized by an NFA.
9. Explain how 7 implies that every regular language is recognized by a DFA.
10. The method just outlined is automatic (you could fairly easily write a computer program to do the translation from regular expression to NFA), and it provides a proof that all regular languages can be recognized by an NFA. But it might produce an NFA with more states than is minimally required. Create an NFA that recognizes $\mathbf{1}^* \cup \mathbf{01}$. Do this two ways:
    a. Following the steps outlined in 1–6.
    b. Any way you like, but using fewer states than in part a.
    c. What is the smallest (fewest states) NFA you can find that recognizes $\mathbf{1}^* \cup \mathbf{01}$?
11. Show that if A is recognized by an NFA, then the complement of A ($\overline{A} = A^c$) is too.

    [Hint: There is an easy way and a hard way – use the easy way.]