3 Finite State Automata

A **finite state automaton** consists of the following:

- a finite set of **states**: S
- a finite **input alphabet**: I
- a **start state**: $s_0 \in S$.
- a set of **accepting states**: $F \subseteq S$
- a **transition function**: $f : S \times I \to S$

Example (M_0) states: $\{A, B, C, D\}$; input alphabet: $\{0, 1\}$; start state: A; accepting states: $\{A, D\}$; transition function described in table below

M_0								
state	A	A	B	B	C	C	D	D
letter	0	1	0	1	0	1	0	1
f	A	B	A	C	A	A	C	B

3.1 Graph Representation

1. It is often easier to visualize what is going on if we represent an automaton with a graph. Create a labeled, directed graph with the following properties:

   - There is a vertex for each state, labeled with the state. (Draw this as a circle with the state inside the circle.
   - There is an edge from state s to state t labeled with letter x if and only if $f(s, x) = t$
   - Accepting states are circled a second time. (We could use shape or color or something else to make it clear which states are accepting states, but double circling is easy.)
   - An extra arrow (not coming from any state) points to the start state. This isn't really an edge in the graph, just an extra bit of labeling.

3.2 Extended transition function

We can extend the transition function to $f^* : S \times I^* \to S$ with the following recursive definition:

- $f^*(s, \lambda) = s$ for any state s
- $f^*(s, xa) = f(f^*(s, x), a)$ for any $x \in I^*$ and $a \in I$

2. Use this definition to determine the following.

   a. $f^*(B, \lambda)$
   b. $f^*(B, 0)$
   c. $f^*(B, 010)$
   d. $f^*(A, 101)$
   e. $f^*(A, 1011)$

3.3 Language Recognition

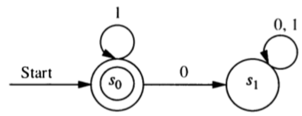
For any automaton M with transition function f, start state s and accepting states F, the language recognized by M (written $L(M)$) is defined as follows:

$$x \in L(M) \iff f^*(s, x) \in F$$

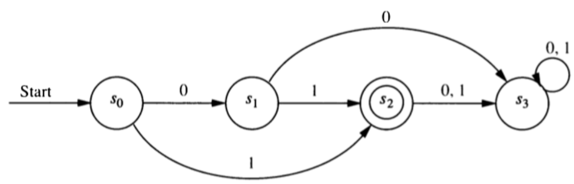
3. For each string x below, compute $f^*(A, x)$. Which of these strings are in $L(M_0)$?
   (We will say that such strings are **accepted by** M_0.)

   a. λ
   b. 0
   c. 1
   d. 010
   e. 1011

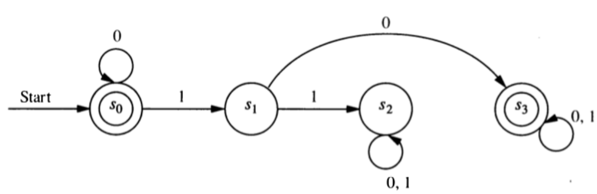
Here are three automata:



M_1



M_2



M_3

4. For each of the machines M_1, M_2, M_3, compute

   a. $f^*(s_0, 10)$
   b. $f^*(s_0, 1011)$
   c. $f^*(s_1, 1011)$

5. For each of the machines M_1, M_2, M_3, determine the language recognized.

6. Create automata that recognize each of the following languages.

   a. The set of bitstrings that begin with two 0's
   b. The set of bitstrings that contain exactly two 0's
   c. The set of bitstrings that contain at least two 0's
   d. The set of bitstrings that contain two consecutive 0's (anywhere in the string)
   e. The set of bitstrings that do not contain two consecutive 0's anywhere
   f. The set of bitstrings that end with two 0's