3 Graph Representations

3.1 4 Representations of Graphs

There are several ways to represent a (simple) graph. The efficiency of some graph algorithms depends on which representation is used to store the graph.

3.1.1 Edge Lists

An edge list simply lists off the edges (i.e., pairs of vertices) in the graph.

3.1.2 Adjacency Lists

For each vertex i, a list of vertices j such that there is an edge from i to j.

3.1.3 Adjacency Matrix

An adjacency matrix A is a square matrix with a row (and column) for each vertex.

- $A_{ij} = 1$ if there is an edge from vertex i to vertex j
- $A_{ij} = 0$ if there is **not** an edge from vertex i to vertex j

3.1.4 Incidence Matrix

An incidence matrix M has a row for each vertex and a column for each edge.

- $M_{ij} = 1$ if edge j is incident with vertex i.
- $M_{ij} = 0$ if edge j is **not** incident with vertex i.

3.2 Representation to Graphs

Draw graphs with the following representations.

1. adjacency list

vertex	adjacent vertices
1	2, 3, 5
2	1
3	1, 4, 5
4	3, 5
5	1, 3, 4

2. edge list: $\{\{1,2\},\{2,3\},\{3,4\},\{4,5\},\{1,3\},\{2,4\},\{3,5\}\}$.

3. adjacency matrix:

```
0 1 1 1
1 0 1 1
1 1 0 0
1 1 0 0
```

4. adjacency matrix:

```
0 1 1 0
1 0 0 1
1 0 0 1
0 1 1 0
```

5. adjacency matrix

```
0 1 0 0 1 1
1 0 1 1 0 1
0 1 0 0 1 1
0 1 0 0 1 0
1 0 1 1 0 1
1 1 1 0 1 0
```

6. incedence matrix

```
1 1 0 0 0 0
0 0 1 1 0 1
0 0 0 0 1 1
1 0 1 0 0 0
0 1 0 1 1 0
```

7. incedence matrix

```
1 1 1 0 0 0 0 0
0 1 1 1 0 1 1 0
0 0 0 1 1 0 0 1
0 0 0 0 0 0 1 1
1 0 0 0 1 1 0 0
```

Variations on the theme

8. Which of these representations can be used (perhaps with slight modification) with **directed graphs**? What adjustments do you need to make?
9. Which of these representations can be used (perhaps with slight modification) for graphs with **multi-edges**? What adjustments do you need to make?
10. Which of these representations can be used (perhaps with slight modification) for graphs with **self-loops**? What adjustments do you need to make?

3.3 Graph to representation

11. Use each representation to represent the following graphs or say why it isn't possible.