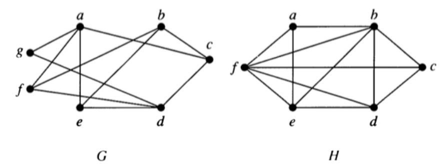
6 Bipartite Graphs

6.1 Definition

A simple graph G is **biparatite** if its vertices can be partitioned into two disjoint subsets V_1 and V_2 and every edge connects a vertex in V_1 with a vertex in V_2. The pair $\langle V_1, V_2 \rangle$ is called a **bipartition** of G.

6.2 Examples

1. Which of the following are bipartite?



2. For what values of n are the following bipartite?

   a. K_n
   b. C_n
   c. Q_n

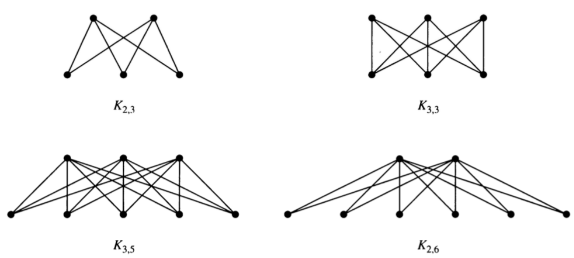
3. Can a bipartite graph have more than one bipartition? If so, give an example. If not, explain why not.

4. Describe an algorithm for checking whether a graph is bipartite. How efficient is your algorithm?

6.3 Complete Bipartite Graphs

A complete biparite graph $K_{m,n}$ has two disjoint sets of vertices V_1 and V_2 with m and n elements. There is an edge between x and y if and only if $x \in V_1$ and $y \in V_2$ or $x \in V_2$ and $y \in V_1$.

Here are some examples.



5. Draw the following graphs: $K_{2,2}$, $K_{4,2}$, $K_{4,3}$

7 Planar and Non-planar Graphs

A graph is a **planar graph** if it can be drawn in the plane with no edge crossings. [Note: You may need to rearrange the vertices and edges to avoid crossings.]

7.1 Euler and Planarity

We have already seen **Euler's Formula for Planar Graphs**:

$$v - e + r = 1 + \text{number of connected components} ,$$

where v is the number of vertices, e the number of edges and r the number of regions.

In particular, for a connected graph we have $v - e + r = 2$.

1. Show that the following are planar.

   a. $K_{2,2}$
   b. $K_{2,3}$
   c. $K_{2,4}$

2. Is $K_{2,n}$ planar for every n?

3. Use Euler's formula to show that K_5 is not planar.

   a. How many vertices does K_5 have?
   b. How many edges does K_5 have?
   c. If K_5 were planar, how many regions would it have?
   d. Show that K_5 can't have that number of regions, so it must not be planar.

4. Use Euler's formula to show that $K_{3,3}$ is not planar.