9 Expected Value

9.1 An Example: GPA

Question: If a student receives 5 A's, 4 B's, and 1 C, what is the student's GPA? (Assume all courses are equally weighted and that an A is worth 4, a B with 3, and a C worth 2.)

1. Explain why $\frac{4+3+2}{3}$ is *not* the correct answer.
2. Show how to correctly calculate the GPA by listing the 10 scores individually.
3. Factor your expression into the following form (fill in the missing numerators):

$$\text{GPA} = 4 \cdot \frac{\quad}{10} + 3 \cdot \frac{\quad}{10} + 2 \cdot \frac{\quad}{10}$$

4. Let X be the random variable that results from randomly selecting a course and recording its grade (on a 4 point scale). Create a probability table for X.
5. How are the numbers in your expression for GPA in #3 related to the numbers in your probability table?

9.2 Generalizing

We can generalize this idea to compute the **mean** (more commonly called **expected value**) of any random variable X. This is denoted either $\text{E}(X)$ or μ_X. E is short for expected value. The Greek letter μ (read "mu") is the Greek version of the letter m for mean. We will use whichever is handier in the moment.

6. Let X be defined by the probability table below. Compute $\text{E}(X)$.

value of X	0	1	2	3
probability	0.2	0.3	0.4	0.1

7. Let H be the number of heads in 3 tosses of a fair coin.
   a. Create a probability table for H
   b. Compute $E(H)$.

8. Use good mathematical notation to write down the definition:

$$\text{E}(X) =$$

9. A raffle has 1000 tickets. Holders of 4 of the tickets get a prize. The other 996 are worth nothing. The four prizes are worth \$500, \$200, \$50, and \$50. Let V be the value of a random raffle ticket.

    a. Create a probabilty table for V.
    b. Compute $\mathrm{E}(V)$.
    c. What does $\mathrm{E}(V)$ tell us about the raffle tickets?

10. Let D be the absolute value of the difference (i.e., higher minus lower) between the values of **two 4-sided dice**.

    a. Create a probability table for D.
    b. Compute $\mathrm{E}(D)$.
    c. What is $p(D = \mathrm{E}(D))$?
    d. What is $p(D < \mathrm{E}(D))$?
    e. What is $p(D > \mathrm{E}(D))$?

We can do these already now, but there are easier ways that take advantage of properties of expected value that we haven't learned yet.

11. In a hand of 5 cards from a standard deck, what is the expected number of diamonds?
12. If you roll 5 standard dice, what is the expected number of 6's?

These two are a bit more challenging, but still doable.

13. In a hand of 5 cards from a standard deck, what is the expected number of suits?
14. If you roll 5 standard dice, what is the expected number of unique numbers rolled?