4 Counting Problems

4.1 Donut problems

The problems below all involved donuts and that numbers 3 and 12. But you will need to know more than that to answer them.

1. Danielle's Donuts sells 12 kinds of donuts. This week they are offering a special: The Mighty Mixer. You get three donuts of three different kinds for $1.99. How many different Might Mixer orders are there?

2. Debra has purchased a Mighty Mixer consisting of one sprinkle donut, one jelly-filled donut, and one pumpkin spice donut. She decides to give them away to some of her friends. She has twelve friends.

   a. How many different ways can she give away the donuts if no one gets more than one?

   b. How many different ways can she give away the donuts if she allows the possibility of giving more than one to the same person?

3. Don's Donuts sells 3 kinds of donuts: plain, chocolate covered, and glazed. You decide to order a different dozen (12) every week until you have ordered every possible dozen. How many weeks will that take?

4. What are the key features that make these situations different from each other? Can you work out a general formula for each of the situations?

5. Write an additional problem of each type (and work out the answer). Do not use the numbers 3 or 12 in your problems. Do not use donuts.

4.2 More Problems

As you do these problems, identify where you are using our counting principles: Multiplication Principle, Addition Principle, Subtraction Principle, Division Principle, Inclusion-Exclusion Principle, Pigeonhole Principle. For problems in the "Donut Family", identify which of the four situations the problem represents.

6. How many bit strings of length eight either start with a 1 bit or end with the two bits 00?

7. How many different ways are there to seat four people around a circular table, where two seatings are considered the same when each person has the same left neighbor and the same right neighbor?

8. A computer company receives 350 applications from computer graduates for a job planning a line of new Web servers. Suppose that 220 of these applicants majored in computer science, 147 majored in business, and 51 majored both in computer science and in business. How many of these applicants majored neither in computer science nor in business?

9. These questions involve a standard deck of cards (52 cards, 13 each are hearts, diamonds, clubs, and spades).

   a. How many cards must be selected to guarantee that at least three of the cards are the same suit?

   b. How many must be selected to guarantee that at least three hearts are selected?

10. What is the least number of area codes needed to guarantee that the 25 million phones in a state can be assigned distinct 10-digit telephone numbers? How many additional phones can be accommodated before needing to add another area code? (Assume that telephone numbers are of the form NXX-NXX-XXXX, where the first three digits form the area code, N represents a digit from 2 to 9 inclusive, and X represents any digit.)

11. Assume that in a group of six people, each pair of individuals consists of two friends or two enemies. Show that there are either three mutual friends or three mutual enemies in the group.

12. Suppose that there are 9 faculty members in the mathematics department and 11 in the computer science department. How many ways are there to select a committee to develop a discrete mathematics course at a school if the committee is to consist of three faculty members from the mathematics department and four from the computer science department?

13. How many bit strings of length 10 contain either five consecutive 0s or five consecutive 1s?