6 Conditional Probability

$p(A \mid B)$ answers the question: Of the times that B happens, how often does A also happen? Common ways this is expressed include

- The probability of A given B
- The probability of A conditional on B
- The probability of A if B
- The probability that A happens when B happens

When $p(B) \neq 0$, then

$$p(A \mid B) = \frac{p(A \cap B)}{p(B)}$$

Note: Usually, $p(A \mid B)$, $p(B \mid A)$ and $p(A \cap B)$ are all different. It is critical to know which of these three applies in a given situation and to use the notation correctly.

6.1 Exercises

A number of school children, some boys and some girls, are asked about their favorite color. The resuls are in the table below.

\	Red	Other
Boy	7	18
Girl	8	12

Suppose we put the names of all the children into a hat and select one randomly. Consider the following events.

- G: The selected child is a girl.
- R: The selected child's favorite color is red.

For each of the following, (a) express the probability in words, (b) determine the probability. For conditional probabilities, compute the probabilities two different ways.

1. $p(R)$

2. $p(\overline{R})$

3. $p(G)$

4. $p(\overline{G})$

5. $p(R \cap G)$

6. $p(R \mid G)$

7. $p(G \mid R)$

8. $p(G \mid \overline{R})$

9. $p(\overline{G} \mid R)$

6.2 Product Rule

Applying a little algebra, we get the following product rule:

$$p(A \cap B) = p(A) \cdot p(B \mid A)$$

6.3 Independence

If $p(A) \neq 0$ and $p(B) \neq 0$ and $p(B \mid A) = p(B)$, then we say that A and B are **independent events**.

6.4 Exercises

10. Show that if $p(B \mid A) = p(B)$, then $p(A \mid B) = p(A)$.

11. The product rule is even simpler when A and B are independent. What is it?

12. Show that $p(A \cap B \cap C) = p(A) \cdot p(B \mid A) \cdot p(C \mid A \cap B)$.

13. Analogous rules hold for intersections of more events as well. Write down the rule for the intersection of 4 events.

14. What is the probability of rolling doubles (two numbers that match) with standard dice? Do this two ways: (a) using the Equally Likely Rule and (b) using the Product Rule.

15. What is the probability of a five-card flush (all cards the same suit)? Do this two ways: (a) using the Equally Likely Rule and (b) using the Product Rule.

16. There are 36 students in Math 252. What is the probability that two people in this class have the same birthday (month and day)? (Hint: Use the Complement Rule.)

    What assumption must we make to do this calculation? Is that a reasonable assumption?

17. If two 6-sided dice are rolled and the first one is a 5, what is the probability that the sum is 10?

18. If two 6-sided dice are rolled and at least one of them is a 5, what is the probability that the sum is 10?

19. If two 6-sided dice are rolled and the sum is 10, what is the probability that at least one of them is a 5?