Elementary subdivisions

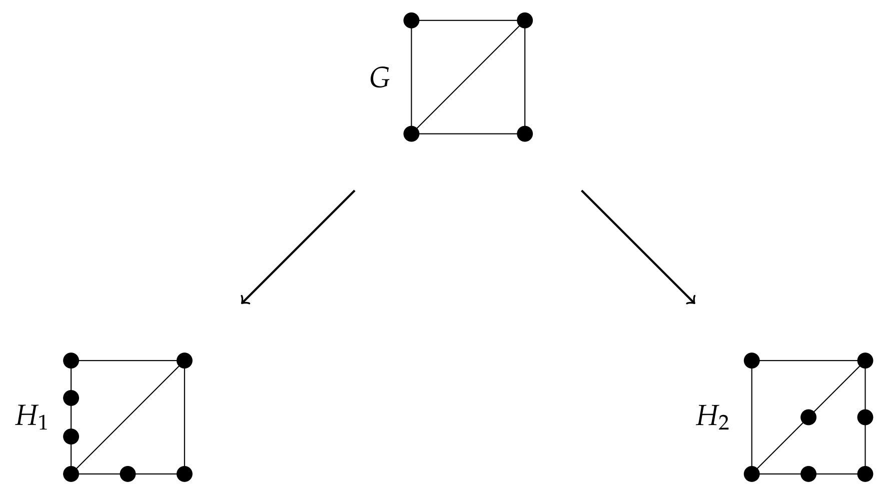
An **elementary subdivision** is basically the addition of a node on an already-existing edge, though if that edge is ”bent” in the process, that is OK.

Draw ”before” and ”after” versions of graphs where an edge $\{a, b\}$ has become two edges $\{a, c\}$, $\{c, b\}$.

Note: Elementary subdivision is a process that preserves planarity (or nonplanarity).

Graph homeomorphism

Start with a graph G (top). Via two different sets of elementary subdivisions, generate graphs H_1 (left) and H_2 (right) from G. H_1 and H_2 are defined to be **homeomorphic**.



Questions:

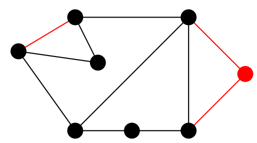
1. Also homeomorphic?

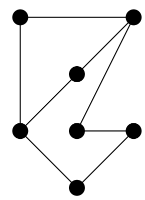
2. Is a graph G homeomorphic to itself?

3. Is H_1 homeomorphic to G?

4. Suppose H_1 and H_2 are homeomorphic. Is it possible to subdivide both H_1 and H_2 in order to obtain isomorphic graphs (i.e., essentially the same graph)? Is the converse true?

Not homeomorphic, but a subgraph is





Kuratowski's Theorem

A graph is nonplanar iff it contains a subgraph that is is homeomorphic to K_5 or to $K_{3,3}$. So, a nonplanar graph contains a copy of K_5 or $K_{3,3}$, possibly with vertices added.